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AN ESTIMATION OF THE PRODUCTION FUNCTION FOR THE MEXICAN ECONOMY, 1980.1 - 2008.4

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Abstract:

We estimated the Cobb-Douglas Production Function augmented by the real exchange rate for the Mexican economy. An homogenous function was obtained. Thus, we claim that the real exchange rate can be taken properly as another production factor. As a derived result, we calculated TFP and therefore the growth accounting for different periods.

(…) Be in your chambers (…), and do not take it amiss if your visitor wears a mask.

"This is indeed a mystery," I remarked, "What do you imagine it means?"

"I have no data yet. It is a capital mistake to theorize before one has data. Insensibly one begins to twist facts to suit theories, instead of theories to suit facts (…)"

Sherlock Holmes to Watson
Sir Arthur Conan Doyle,
A Scandal in Bohemia

1. Introduction

The historical lack of official series of capital stocks in Mexico has been a serious obstacle to estimate the traditional Cobb-Douglas Production Function. Therefore, economic growth analysis derived from the supply side has been very limited.

Loria and de Jesus (2007) applied the Perpetual Inventory Method-Adjusted by a non-fixed depreciation factor and by so were able to develop a very handy methodology to calculate this important variable for quarterly and annual data. We argue that by having a coherent series of the capital stock it might be possible to estimate a production function by using the CVAR procedure, trying to see what the data say through a well specified model.

But we also pursued another objective, which is to incorporate the bi-lateral real exchange rate (Mexican Peso-US Dollar) as an additional production factor, due to the huge dependence of Mexico in commerce and finance to the US economy, and since the Mexican economy has a high degree of openness after the mid 80's.

Traditionally, nominal and real depreciations have had at the same time real and inflationary effects. According to economic analysis and to what history report, we might fairly claim that in the short run depreciations have –at the same time- contractionary effects on output, labor and capital as well as important inflationary pressures, taking the whole economy to dark economic episodes. But according to economic theory (Marshall-

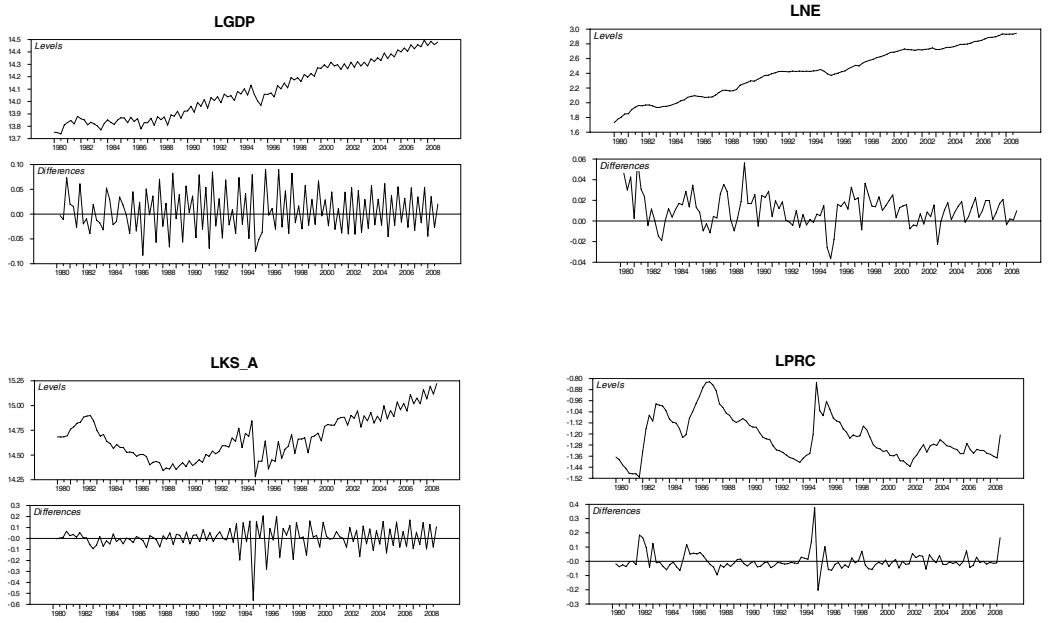
Lerner Condition), exchange depreciations must carry on to output expansions in the long run.

In this work we focus our interest exclusively in the long run relations, trying to prove the above mentioned theory. In to do so we used the CVAR approach.

We claim that it is possible to estimate a homogeneous production function, which in turns allow us to do economic and statistical introspection and to decompose the growth accounting.

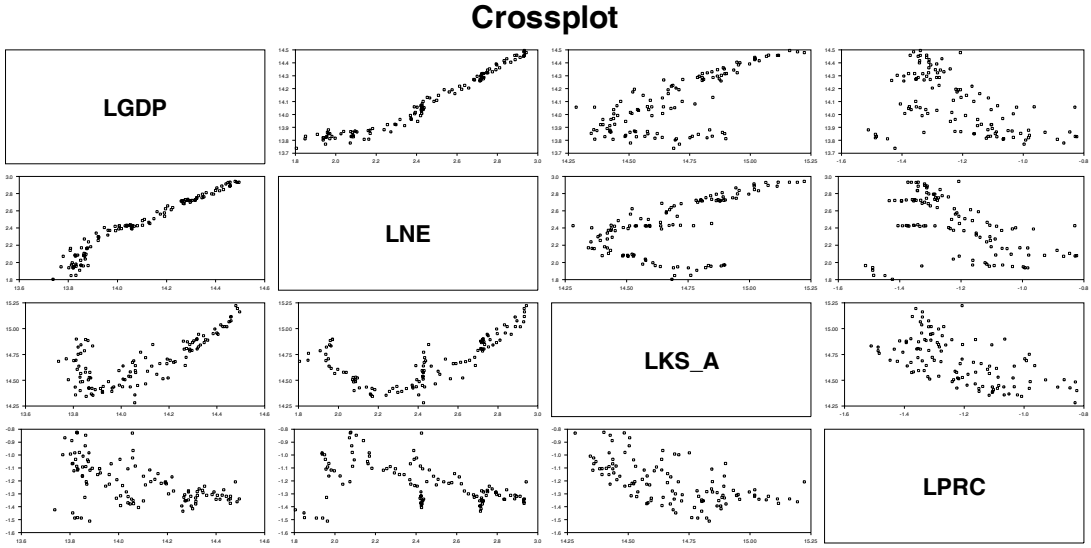
2. The data

According to the "Allowing the Data to Speak Freely Approach" (Hoover 2007, et al) it is convenient to look first at the raw data.



It is apparent the high seasonality and multiple shocks in output, labor and capital, and the cyclical behavior of the real exchange rate on time. Also, it is very clear that besides the multiple shocks, there is a huge shift -probably in the trend- in 1995:1.

A cross plot of the variables combined suggests, at this point, possible economic and statistic relations among variables.



A very clear (positive) relation arises between output (gdp) and labor (ne) as expected, which is also stated by the economic theory, either classic or keynesian. The same can be argued, but not with the same strength, between gdp and capital stocks (ks_a). But, so far, we cannot say the same for the rest of the empirical economic relations. Statistical inference is required.

It is also important to highlight the cyclical and erratic behavior of the real exchange rate that at this point suggests that it might be exogenously participating in the cointegrating space of the system.

3. Specification

Given our main objectives, we defined the information vector that accounts for the basis of the over all econometric procedure:

x' = [(gdp, ne, ks_a, prc, shift dummy(1995:1), permanent dummies)] (1)

Variables in lower case represent logs.

To find a relationship that adequately represents the data generating process, it is necessary to find a long-term balanced equation among the defined variables. We estimated a CVAR model trying to include the mayor insights of the methodology and techniques described in Juselius (2006). The following expression represents the unrestricted VAR model:

$$X_t = \Pi_1 X_{t-1} + \Pi_2 X_{t-2} + ,,, + \Pi_k X_{t-k} + \Phi D_t + \varepsilon_t \quad (2)$$

Where Π_i represents the matrix coefficients

Equation (2) can be expressed in the Vector Error Correction Model (VECM) form:

$$\Delta X_t = \Gamma(L)\Delta X_t + \Pi X_{t-1} + \Phi D_t + \varepsilon_t \quad (3)$$

Where (L) is the lag operator, $\Gamma_i = \Pi_{i+1} + \Pi_{i+2} + ... + \Pi_k$, $\Pi = -(\Pi_1 + \Pi_2 + ... + \Pi_k)$. In order to achieve a correct basic specification it is compulsory that the innovations of the unrestricted VAR follow a Gaussian distribution: $\varepsilon_t \sim IN(0, \Omega)$ and passes all the misspecifications tests (see the appendix). We found that a good specification might be the following:

- 2 lags
- To account for the erratic evolution of the main four variables, we imposed four permanent dummies: Dum8102, Dum8401, Dum8901, Dum9703; and one shift (level) dummy (1995:1) that accounts for the previous major financial crisis "efecto tequila" and the intensification of economic integration derived from the implementation of NAFTA afterwards.
- After deliberations we decided to take prc exogenously since its dynamics seem to follow a very complicated process. In effect, due to the importance of this variable to the overall economy, the economic authority has considered it sometimes as an instrument of stability, in some other span times as an instrument of competitiveness (during the 80's), and since the late 90's to target inflation.

  By defining prc exogenous we were able to reduce the number of dummies (from 13 to 4) and to incorporate information in a more direct fashion. With 3 endogenous variables we found full rank, but according to the recursive tests and

by analyzing the graphs of the concentrated model, finally we decided to reduce it to r = 2. We also decided to normalize on gdp and on ks_a, since –at this point- ne does not appear to be correcting for disequilibria in the short run (alphas) and since the test of weak exogeneity was not rejected.

- The exclusion test suggests that all the variables above defined should be included in the estimation of the cointegrating space. The sequence of the above mentioned tests is developed in the appendix.

4. The identified model and econometric issues

Once all these tests passed the standard criteria on the unrestricted VAR(2), we proceeded to identify the beta matrix for the VECM.

After several identification tests on the two cointegrating vectors, we found that the final model with high economic sense regarding our main objectives and the previous statistical results should be defined in terms of output and capital. Labor –in all the previous cases- resulted to be weakly exogenous and by forcing it to be endogenous – even though we could get statistical identification- it did not deliver any robust economic meaning for the whole system.

It is worth mentioning that trying with different specifications, we obtained identification and homogeneity for the elasticities of capital and labor, given the statistical significance and the positive sign of the elasticity of the real exchange rate to output.

Therefore, by imposing two restrictions to the first vector (gdp) and three to the second one (ks_a) we obtained an over-identified system <R(i.j); (1.2): 1; (2.1): 2> with the following results:

TEST OF RESTRICTED MODEL: CHISQR(3) = 6.447 [0.092]

RE-NORMALIZATION OF THE EIGENVECTORS:

THE EIGENVECTOR(s)(transposed)

	LGDP	LNE	LKS_A	LPRC	C(1995:01)	TREND
Beta(1)	84.917	-40.251	-31.064	-13.602	-3.366	0.000
Beta(2)	-7.553	0.000	12.261	16.894	0.000	0.000

THE MATRICES BASED ON 2 COINTEGRATING VECTORS:

BETA(transposed)

	LGDP	LNE	LKS_A	LPRC	C(1995:01)	TREND
Beta(1)	1.000	**-0.474**	**-0.366**	**-0.160**	-0.040	0.000
	(.NA)	(-40.931)	(-32.385)	(-12.643)	(-6.012)	(.NA)
Beta(2)	-0.616	0.000	1.000	1.378	0.000	0.000
	(-4.629)	(.NA)	(.NA)	(8.574)	(.NA)	(.NA)

ALPHA

	Alpha(1)	Alpha(2)
DLGDP	-0.246	-0.049
	(-1.973)	(-2.737)
DLNE	0.178	0.028
	(2.929)	(3.170)
DLKS_A	0.971	-0.004
	(2.921)	(-0.081)

PI

	LGDP	LNE	LKS_A	LPRC	C(1995:01)	TREND
DLGDP	-0.216	0.117	0.041	-0.029	0.010	0.000
	(-1.879)	(1.973)	(1.339)	(-2.600)	(1.973)	(.NA)
DLNE	0.161	-0.085	-0.037	0.010	-0.007	0.000
	(2.875)	(-2.929)	(-2.517)	(1.843)	(-2.929)	(.NA)
DLKS_A	0.973	-0.460	-0.359	-0.161	-0.038	0.000
	(3.182)	(-2.921)	(-4.437)	(-5.490)	(-2.921)	(.NA)

Log-Likelihood = 1414.951

It is to be noticed that even though we restricted labor in the identification for the second vector, according to the PI matrix it is reactioning to most of the variables. According to the alphas, all the variables are adjusting for both equations, but capital only to the first. After identifying the system, weak exogeneity was rejected in all cases. So, to this point it was not that clear that labor is the pulling force, as supposed before. But by performing the MA representation, in all the cases labor and gdp resulted to move the system.

Normalizing the Loadings to the Common Trends, we obtained that labor and gdp were significant.

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THE MA-REPRESENTATION AND DECOMPOSITION OF THE TREND

The Coefficients of the Common Trends:
RE-NORMALIZATION OF ALPHA Orthogonal:

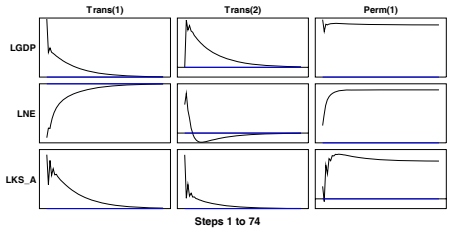
ALPHA Orthogonal (transposed)
       LGDP   LNE   LKS_A
CT(1) -0.494 -0.869 0.034

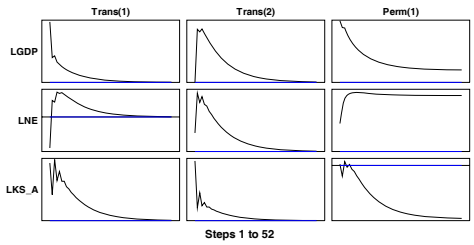
ALPHA Orthogonal (transposed)
       LGDP    LNE   LKS_A
CT(1)  0.568  1.000  -0.040
      (1.710) (.NA) (-0.741)

The Loadings to the Common Trends, BETA_ORT(tilde):
        CT1
LGDP   0.868
      (4.204)
LNE    1.419
      (4.204)
LKS_A  0.535
      (4.204)

The Long-Run Impact Matrix, C
       LGDP     LNE    LKS_A
LGDP   0.493   0.868   -0.034
      (2.384) (4.204) (-0.751)
LNE    0.806   1.419   -0.056
      (2.384) (4.204) (-0.751)
LKS_A  0.304   0.535   -0.021
      (2.384) (4.204) (-0.751)
```

Finally, we performed the Structural MA Model, keeping the original restrictions on beta. It is difficult to interpret these pictures since we know that by definition, the residuals are not properly identified and are very sensitive to the restrictions, as can be seeing in the second picture, in which we deleted them.





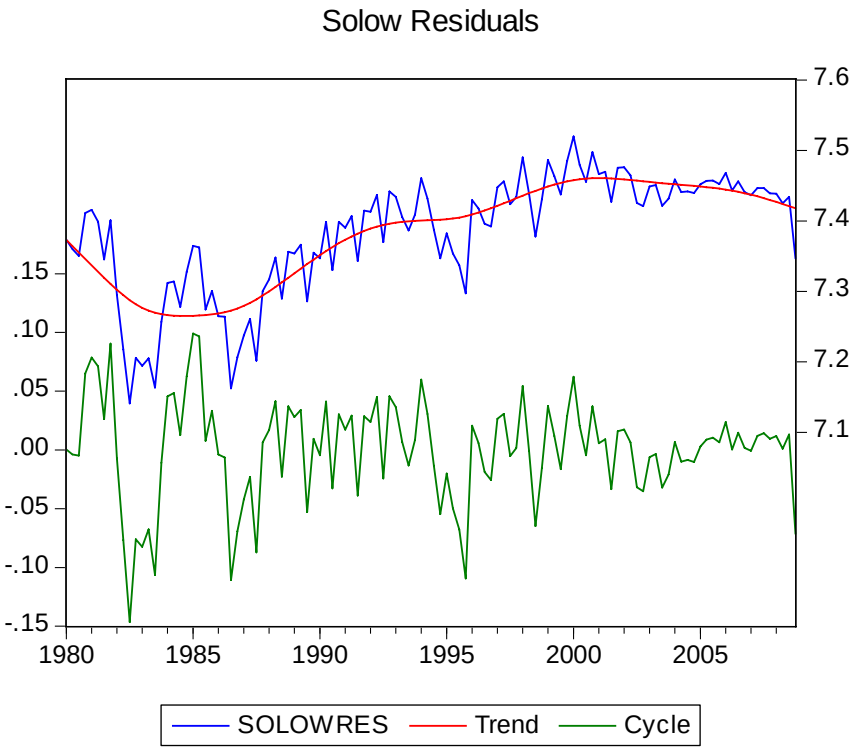
5. Economic analysis

We obtained a system that proved to be stationary and homogeneous on the parameters of the Cobb-Douglas production function augmented by the real exchange rate, since the sum of the three elasticities is 1.

It is also to be noticed the importance of real exchange rate in the long run growth (0.16) and the slight difference in favor of labor (0.474 vs 0.366), as corresponds to a labor intensive country such as Mexico. Nevertheless, it would have been expected that the elasticity of labor to be higher. This point requires a further analysis that is not accomplished here.

The second equation suggests that the capital stock is very sensitive to prc and secondly to gdp. In effect, depreciations shrink seriously capital stocks imports and domestic capital accumulation and economic growth have the opposite effects, but in a more moderated manner.

Altogether, the alphas suggest adjusting process to the stationary relations. Derived from the estimated output, we calculated TFP (a).



Other very important and useful tool of the production function is that it allows calculating the growth accounting, which resulted to be the following for selected periods:

Period	*y*	*k*	*l*	*q*	*a*
1980-2008	2.55	0.62	1.93	0.05	-0.04
1988-2000	3.66	1.21	2.12	-0.38	0.71
1988-2008	3.10	1.46	1.80	-0.22	0.06

This last table suggests that economic growth for the different periods has been basically explained by labor and very little by TFP, which shows to be negative for the whole historical period (-0.04%). This could explain the main cause of the slow rate of growth of the last decade.

6. Appendix

TESTS OF INCORRECT ESPECIFICATION

1. Unrestricted VAR(2)

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DUMMY VARIABLES:
       DUM8102P{0} DUM8401P{0} DUM8901P{0} DUM9703P{0}
DLGDP       0.006       0.061       0.011       0.000
           (0.405)     (3.878)     (0.751)     (0.021)
DLNE        0.038       0.019       0.052      -0.029
           (4.520)     (2.232)     (6.371)    (-3.552)
DLKS_A      0.013       0.048       0.022       0.053
           (0.297)     (1.089)     (0.500)     (1.221)

CENTERED SEASONALS
         SEAS1    SEAS2   SEAS3
DLGDP    -0.054   0.016   -0.033
       (-10.036) (2.954) (-4.278)
DLNE     -0.011   0.008   -0.029
        (-3.865) (2.506) (-6.932)
DLKS_A    0.024   0.062   -0.013
         (1.591) (3.931) (-0.602)

CONSTANT

DLGDP    5.051
        (4.351)
DLNE    -0.598
       (-0.949)
DLKS_A  -7.685
       (-2.322)
```

Autocorrelation
LM(1): ChiSqr(9) = 6.183 [0.721]
LM(2): ChiSqr(9) = 17.471 [0.042]

Test for Normality: ChiSqr(6) = 7.556 [0.272]

Test for ARCH:
LM(1): ChiSqr(36) = 27.686 [0.838]
LM(2): ChiSqr(72) = 57.243 [0.898]

Weak Exogeneity

r	DGF	5%C.V.	LGDP	LNE	LKS_A
1	1	3.841	1.849	0.283	6.243
			[0.174]	[0.595]	[0.012]
2	2	5.991	10.198	0.494	18.823
			[0.006]	**[0.781]**	[0.000]

Test of Exclusion

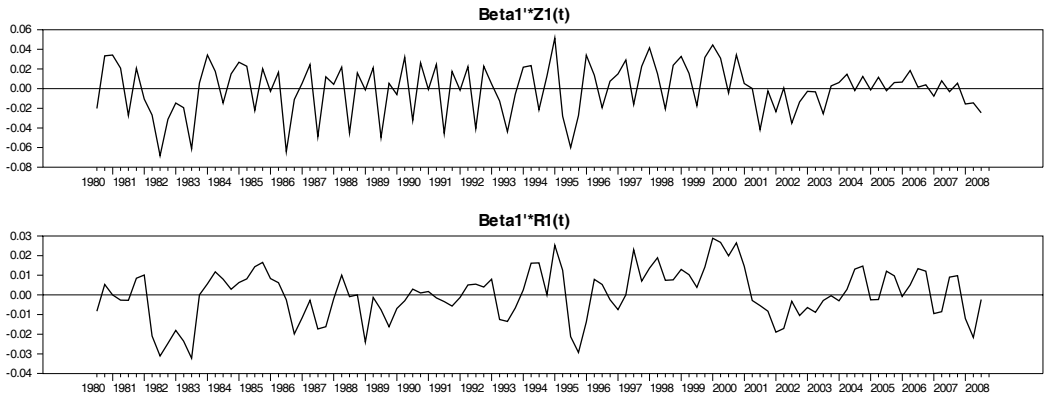
LR-test, Chi-Square(r), P-values in brackets.

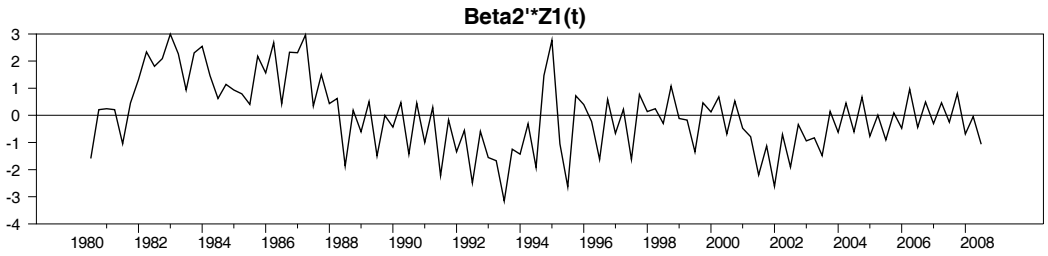
r	DGF	5% C.V.	LGDP	LNE	LKS_A	LPRC	C(1995:01)	TREND
1	1	3.841	11.207	10.957	13.025	0.048	9.388	3.066
			[0.001]	[0.001]	[0.000]	[0.827]	[0.002]	[0.080]
2	2	5.991	23.783	12.394	23.438	11.187	11.129	6.212
			[0.000]	**[0.002]**	**[0.000]**	**[0.004]**	**[0.004]**	**[0.045]**

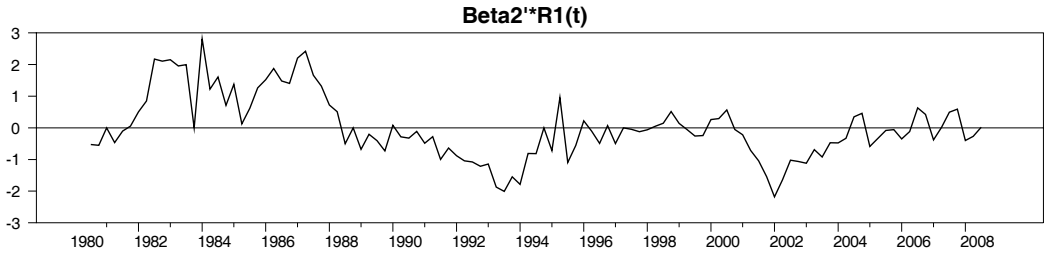
Lag length

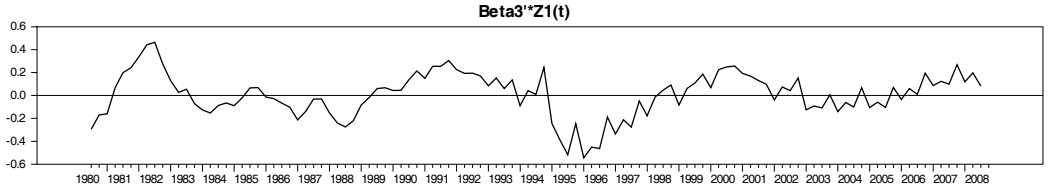
Model	k	T	Regr	Log-Lik	SC	H-Q	LM(1)	LM(k)
VAR(5)	5	111	32	1451.800	-22.085	-23.478	0.015	0.151
VAR(4)	4	111	28	1440.489	-22.391	-23.609	0.038	0.225
VAR(3)	3	111	24	1411.805	-22.383	-23.428	0.000	0.098
VAR(2)	**2**	**111**	**20**	**1397.900**	**-22.642**	**-23.512**	**0.662**	**0.062**
VAR(1)	1	111	16	1326.721	-21.868	-22.565	0.000	0.000

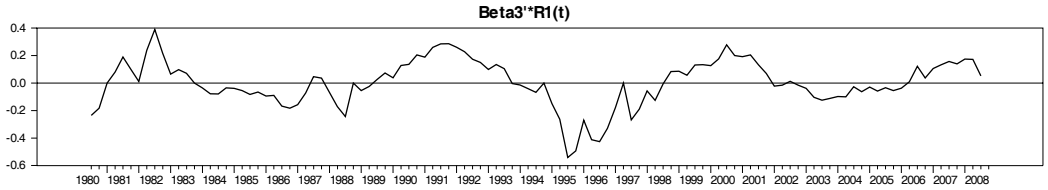
Cointegrating relations

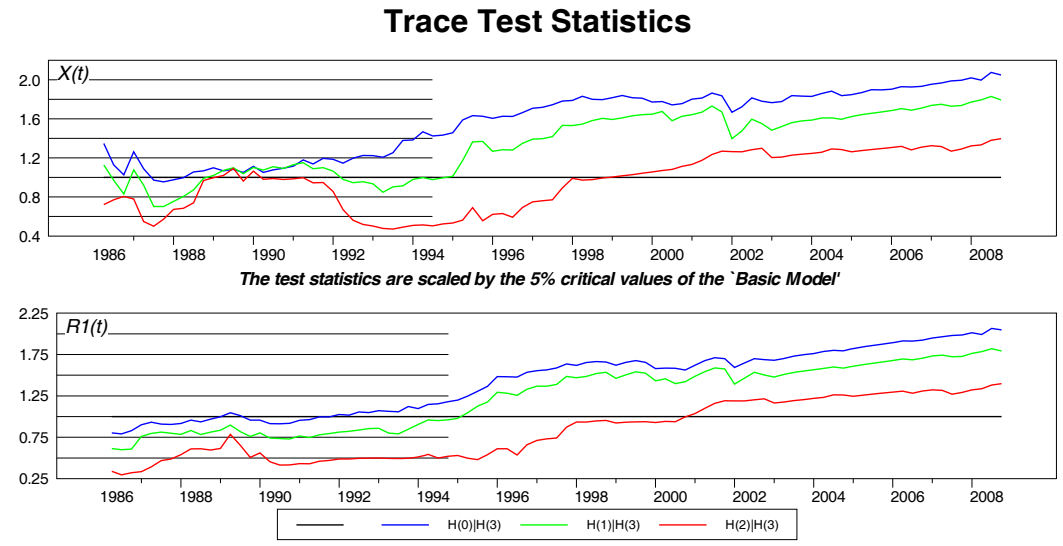


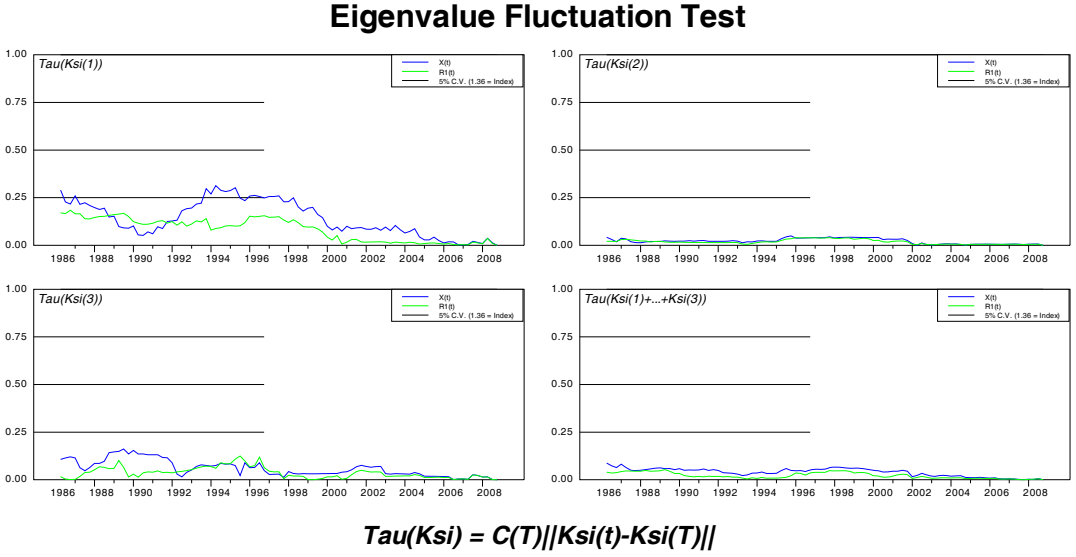

Beta2'*Z1(t)
3
2
1
0
-1
-2
-3
-4
1980 1982 1984 1986 1988 1990 1992 1994 1996 1998 2000 2002 2004 2006 2008


Beta2'*R1(t)
3
2
1
0
-1
-2
-3
1980 1982 1984 1986 1988 1990 1992 1994 1996 1998 2000 2002 2004 2006 2008


Beta3'*Z1(t)
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
1980 1981 1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008


Beta3'*R1(t)
0.4
0.2
0.0
-0.2
-0.4
-0.6
1980 1981 1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008





According to the two previous pictures, it seems convenient to remove the third cointegrating relation.

2. Tests of the VECM

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THE MA-REPRESENTATION AND DECOMPOSITION OF THE TREND (Restricted Model)

The Coefficients of the Common Trends:

RE-NORMALIZATION OF ALPHA Orthogonal:

ALPHA Orthogonal (transposed)
       LGDP   LNE   LKS_A
CT(1) -0.494 -0.869 0.034

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      (2.384) (4.204) (-0.751)
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